

DEVELOPMENT OF A DECISION SUPPORT SYSTEM FOR SOLAR PANEL SELECTION USING INTEGRATED ANALYTIC HIERARCHY PROCESS AND GREY RELATIONAL ANALYSIS

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ABSTRACT

The solar panel is an important component of a photovoltaic system and there has been a lot of research works carried out in order to reduce its material costs while improving energy efficiency. Solar panel Selection for PV system is a Multi-Criteria Decision Making (MCDM) problem as it involves both qualitative and quantitative criteria. In order to select the best solar panel, it is necessary to make a balance between tangible and intangible criteria that conflict with each other. The aim of the paper is to develop an integrated approach based on Analytical Hierarchy Process (AHP) and Grey Relational Analysis (GRA) for a 100W solar panel selection. The additive normalization method is used as the prioritization method to calculate the priority vector. Also a computer program is developed in Excel to calculate the mathematical analysis and obtain the results.

KEYWORDS: Solar Panel, Analytical Hierarchy Process, Grey Relational Analysis & MCDM

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INTRODUCTION

In recent years, Solar panels are used for less scale energy generation, particularly for commercial or residential use in complexes or individual buildings with efficiency ranges from 18%-12%. There are two different crystal types used in solar panels such as polycrystalline and monocrystalline (Hamilton J, 2011). Polycrystalline panels are usually less efficient due to the presence of only one crystal, but are cheaper while monocrystalline is a small piece more expensive, but generally more efficient (Jager W, 2006). The total cost of the solar panel depends on size (in W), the brand, the physical size, the longevity /durability and any certifications of the panel.

In this study, a solar panel assessment system is developed, which includes electrical, mechanical, economic, customer and environment criteria. The comparative analysis of different solar panel brands is made by using the AHP. Among selected popular solar panel brands for 100W, the best solar panel selection is obtained by evaluating comprehensively. This paper proposes an integrated Solar panel selection model based on the integration of Analytical Hierarchy Process (AHP) and Grey Relational Analysis (GRA). AHP is used to systematically combine judgments from different evaluators and obtain the weights of qualitative criteria. The relative weights of criteria are determined and these weights are used as coefficients of GRA model. The GRA algorithm is used to combine the qualitative and quantitative data, considering its characteristics of “large-is-better” or “smaller-is-better” to perform the evaluation of Solar panel selection. To make the calculation procedure simple and to obtain

the results accurately, a decision support system has been developed using Excel. Finally, the effectiveness of this integrated model is validated through the data collected from leading solar panel companies in India.

REVIEW ON AHP AND MCDM METHODS

In the field of renewable energy, utility based models like MAUT, AHP, Weighted Sum Method, and Weighted Product Method are mostly preferred for ranking energy technologies. AHP and MAUT have been used in alternative electricity supply strategies (Dyer 1990). Ramanathan R, et al., (1995) developed an integrated model using goal programming and AHP for Energy resource allocation incorporating qualitative and quantitative criteria. AHP has emerged as a better method in making decisions that include risk, social factor. Energy storage, power quality issues, energy allocation, optimal dispatch, sustainability in the field of renewable energy are some of the important criteria in the segment of energy planning. William Ho (2007) reviewed sixty six different articles written between 1997 and 2006, concerning the applications of integrated AHPs in decision making. This study examined five tools that commonly combined with the AHP include mathematical programming, quality function deployment (QFD), meta-heuristics, SWOT analysis, and data envelopment analysis (DEA). Kuo et al (2002) developed a decision support system for locating a new convenience store. Janjic et al. (2012) have discussed criteria like production, cost and other constraints for distributed generation using optimal dispatch. H. Meyar-Naim, et al., (2012) presented a case study of Iran, which used AHP to assess power generation system from a sustainability perspective. A. Toossi et al., (2013) developed an AHP based decision model for energy systems policy making. H. Aras Ş et al., (2004) used AHP to select a wind power station in a campus of university resulting in evaluation of criteria with topography and security as most important. Y.-y. Wu et al., (2011) discussed on AHP with other MCDM techniques as well like Goal Programming (GP) and fuzzy logic. Barin et al., (2009) studied the use of AHP and fuzzy logic for selection of storage energy technologies with concern in power quality context considering efficiency, load management, technical maturity, cost, and life-cycle as criteria.

Based on the review of literature, it is clear that LW models do not include quantitative factors (such as cost and delivery), whereas the MP models have the disadvantage of not including qualitative factors (such as finance and customer service). Many papers used AHP method to deal with the solar panel selection problem, most of them using the qualitative criteria in evaluation. However, in this paper, not only the qualitative criteria, but quantitative criteria are considered simultaneously in solar panel performance evaluation and selection. Additionally, the quantitative criteria had the characteristic of “smaller-is-better” and the “larger-is-better”.

GREY RELATIONAL ANALYSIS

Grey system theory was originated with Deng (1982). A system having incomplete information is called Grey system. It can be used to solve ambiguous problems in cases with discrete data and incomplete information (Deng, 1989). It is, therefore, a theory and methodology that deals with poor, incomplete, or uncertain systematic problems. One of the major advantages of Grey system theory is that it can generate satisfactory outcomes using a relatively small amount of data or with great variability in factors. It achieves this by increasing the regularity of data with proper treatment (Li et al., 1997). Like fuzzy set theory, Grey theory is thus an effective mathematical model for resolving uncertain and indeterminate problems (Hsu and Wen, 2000).

Grey relational analysis is a method to analyze the relational grade for discrete sequences. This is unlike the traditional statistical analysis, handling the relation between variables. Some of its defects are: (1) it must have plenty of

data; (2) data distribution must be typical; (3) a few factors are allowed and can be expressed functionally. But the Grey relational analysis requires less data and can analyze many factors that can overcome the disadvantages of statistics method (Chih-Hung Tsai et al, 2003).

The concept of Gray relational space was proposed by Deng (1982) on the basis of the combined concepts of system theory, space theory, and control theory. It can be used to capture the correlations between the reference factors and other compared factors of a system (Deng, 1988). GRA analyses uncertain relations between one main factor and all the other factors in a given system (Liang, 1999). Fields covered by Grey theory include forecasting, system control, data-processing, modelling, and decision making (Chang et al., 1996; Hsu and Wen, 2000). GRA has been successfully applied to various decision problems – including performance evaluation of airlines (Feng and Wang, 2000), generation scheduling of hydroelectricity (Liang, 1999), image compression (Hsu et al., 2000), and multiple-attribute decision-making problems (Wu, 2003; Chiou and Tzeng, 2001). The details of the GRA are summarized in Appendix.

CASE STUDY

The present research proposes an integrated model for solar panel selection. In this case study, a solar panel assessment system is composed, which includes electrical, mechanical, economic and customer satisfaction criteria by combining AHP and GRA into a single evaluation model for establishing a decision support system. The application of the integrated model includes four steps, as shown in Figure 1.

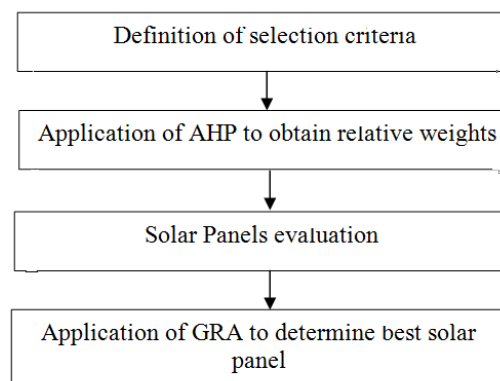


Figure 1: Steps Involved in the Integrated Model for Solar Panel Selection

Defining the Problem

One of the most significant decisions in the photovoltaic system design is the selection of these solar panels. There are four potential solar panels (SP_1 to SP_4) are considered in this case study and the following criteria and sub-criteria considered for selection:

- Electrical Properties (C_1): Electrical properties of the are the Major Criteria which is Determined by the following Sub-Criteria:
 - PTC Power Rating (C_{11}): It is expressed in Watts. The peak power rating is the maximum output under standard test conditions and higher power rating is preferred.
 - Temperature co-efficient (C_{12}): The temperature co-efficient rating is important to determine what the

impact heat has on a solar panel's operation after installation. The lower the percentage per degree Celsius, the better.

- Peak efficiency (C_{13}): with high peak efficiency is preferred.
- Energy density (C_{14}): It is expressed in amps and high value of power current is desirable. Its notation is C_{14} .
- STC Power (C_{15}): It is expressed in volts and high value of power current is desirable. Its notation is C_{15} .
- Weight (C_2): with less weight is preferable.
- Quality: The quality of different solar panels. Its notation is C_3 .
- Service support (C_4)

The main objective of this research is to choose best solar panel based on the above mentioned tangible and tangible criteria that will meet all the needs of the customers. The hierarchy for the selection of the best solar panel is depicted in Figure 3.

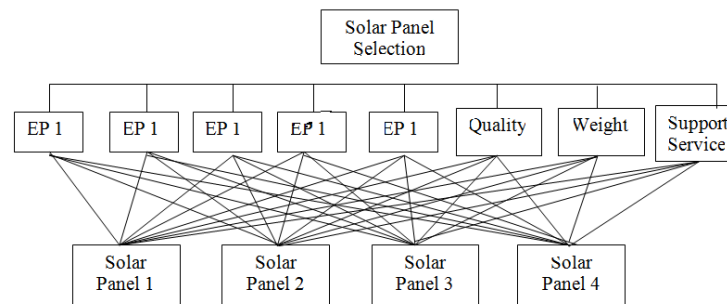


Figure 3: The Hierarchical Structure for Solar Panel Selection (Both Qualitative and Quantitative)

EP 1- PTC Power rating

EP 2- Temperature co-efficient

EP 3- Peak efficiency

EP 4- Energy Density

EP 5- STC Power

Using AHP to Obtain Relative Weights of Criteria

The AHP evaluation process was conducted after interviewing three department managers of purchasing, quality assurance, and engineering. A program has been developed to aggregate evaluators' judgments and calculates the weights using EXCEL. The judgment matrix size is 8x8. The consistency ratio of the pairwise comparison, judgment matrix was $0.05 < 0.1$. This indicated that the judgment matrix had satisfactory consistency. Various firms offer different weights to different criteria. The present study used AHP to determine the relative weight of each criterion. A Nine-point rating scale was designed as shown in Table 1 and priority weights of the criteria were determined using pair wise comparisons.

Table 1: Saaty's 9 Point Scale Of Relative Importance

Intensity of relative importance	Definition
1	equal importance
3	weak importance of one over the other
5	strong importance
7	demonstrated importance over the other
9	absolute importance
2,4,6,8	Intermediate values between

The weights of the evaluative criteria are presented in Table 2. It can be seen that PTC Power Rating was considered to be the most important criterion in the solar panel selection process.

Table 2: Relative Weight of Qualitative Criteria

Criteria	PTC	Peak Efficiency	Temp Co Efficient	Weight	Quality	Service Support	STC Power	Energy Density
Weight	0.27	0.2	0.16	0.1	0.1	0.02	0.09	0.06

From the Table 2, it can be seen that PTC was considered to be the most important criterion in the solar panel selection process. Qualitative and quantitative evaluation of candidate solar panels is shown in Table 3. A qualitative ranking was given by a review committee, then convert these qualitative ranking into a rating scale of 1 to 5 (larger-the-better).

Table 3: Qualitative and Quantitative Evaluation of Candidate Solar Panels

Solar Panel	PTC	Peak Efficiency	Temp Co Efficient	Weight	Quality	Service Support	STC Power	Energy Density
1	174	14.2	0.49	15.3	4.3	4.01	141	11.3
2	179	15.6	0.38	14.6	4	3.8	155.1	12.3
3	180	14.3	0.5	35	3.9	4	135.4	11.8
4	185.9	16	0.32	18	4	3.9	162	13

Evaluation of Potential Solar Panels

The qualitative and quantitative data are shown in Table 3. To demonstrate the GRA model in this solar panels selection problem, the solar panels were considered as alternatives i ($i = 1, 2, 3$ and 4), whereas evaluative factors were viewed as criteria j for each alternative ($j = 1, 2, 3, 4, 5, 6, 7$ and 8). The higher the weighted value of each qualitative criterion, the greater its capability to support the service. Low weight, low energy density and low temperature Co-efficient are certainly advantages to the customers. The qualitative data, PTC, Peak efficiency, quality and service support therefore, had the characteristic of "larger-is-better". In the case of quantitative data, weight, energy density and temperature Co-efficient were "smaller-is-better".

Using GRA to Determine Best Solar Panel

This procedure used GRA to integrate the qualitative evaluation from three evaluators using AHP and quantitative data:

Generating Referential Series of X_0

The qualitative and quantitative evaluation of four potential solar panels can be represented as a matrix as follows:

$$X_i = \begin{bmatrix} 174 & 14.2 & 0.49 & 15.3 & 4.3 & 4.01 & 141 & 11.3 \\ 179 & 15.6 & 0.38 & 14.6 & 4 & 3.8 & 155.1 & 12.3 \\ 180 & 14.3 & 0.50 & 35 & 3.9 & 4.0 & 135.4 & 11.8 \\ 185.9 & 16 & 0.32 & 18 & 4.0 & 3.9 & 162 & 13 \end{bmatrix}$$

Then the referential series of x_0 is (185.9, 16, 0.32, 14.6, 4.3, 4.01, 162, 11.3) and the compared series of x_1 is (174, 14.2, 0.49, 15.3, 4.3, 4.01, 141, 11.3)

$$x_2 = (179, 15.6, 0.38, 14.6, 4, 3.8, 155.1, 12.3)$$

$$x_3 = (180, 14.3, 0.50, 35, 3.9, 4.0, 135.4, 11.8) \text{ and}$$

$$x_4 = (185.9, 16, 0.32, 18, 4.0, 3.9, 162, 13)$$

Normalizing Data Set

The series data in this case can be treated using two approaches: larger-is-better and smaller-is-better. Therefore, the normalized referential series of x_0 becomes x_0^* : The “larger-is-better” data transformation of equation (1) in Appendix is applied to qualitative criteria and whereas the “smaller-is-better” criteria set of “weight”, “energy density” and “temperature Co-efficient” are transformed using equation (2). The calculations are shown below. According to the calculation results, the referential series of x_0^* becomes (1, 1, 1, 1):

$$x_1^*(1) = 174 - 174 / 185.9 - 174 = 0$$

$$x_2^*(1) = 179 - 174 / 185.9 - 174 = 0.420$$

$$x_3^*(1) = 180 - 174 / 185.9 - 174 = 0.5042$$

$$x_4^*(1) = 185.9 - 174 / 185.9 - 174 = 1$$

The same procedure is applied to calculate the remaining values of normalization.

Calculating the Grey Relational Coefficient $\beta_{0i}(j)$

This step is to use the grey relational equation to calculate the grey relational coefficient by using equation (3). In this paper, the distinguished coefficient was set at 0.5.

$$\text{For example, } \beta_{01}(1) = (0 + 0.5 \times 1) / (1 + 0.5 \times 1) = 0.3333$$

$$\beta_{02}(1) = (0 + 0.5 \times 1) / (0.5798 + 0.5 \times 1) = 0.4630$$

$$\beta_{03}(1) = (0 + 0.5 \times 1) / (0.4958 + 0.5 \times 1) = 0.5021$$

$$\beta_{04}(1) = (0 + 0.5 \times 1) / (0 + 0.5 \times 1) = 1$$

The remaining values of $\beta_{0i}(j)$ are calculated in the similar way as explained above and the calculation results are shown in Table 4.

Table 4: The Values of Grey Relational Coefficient

	PTC	Peak Efficiency	Temp Co Efficient	Weight	Quality	Service Support	STC Power	Energy Density
Weight	0.27	0.2	0.16	0.1	0.1	0.02	0.09	0.06
Solar panel 1	0.3333	0.3333	0.3461	0.9357	1	1	0.3877	1
Solar panel 2	0.4630	0.6923	0.6	1	0.4	0.3333	0.6584	0.4594
Solar panel 3	0.5021	0.3461	0.3333	0.3333	0.3333	0.9130	0.3333	0.6296
Solar panel 4	1	1	1	0.75	0.4	0.4883	1	0.3333

Calculating Degree of Grey Equation Coefficient η_{0i}

The Grey relational grade corresponds to the correlation between two series. It is not important in a decision-making. Rather, the ranking order of the relational grade is the most important information. The final step is to calculate the grade of the grey relational coefficient η_{0i} by using equation (4). According to GRA, the alternative with the highest grey relational grade is the most important (or optimal) alternative (Wu and Chen, 1999). Therefore, in this study, the priorities of potential solar panels can be ranked in accordance with the grey relational grade values – because the relative weights (W_i) of evaluative criteria are determined using AHP. After obtaining the Grey relational coefficient, the average of the Grey relational coefficient is taken as the Grey relational grade. The result of the Grey relational grade calculation for each solar panel is shown below:

$$\eta_{01} = (0.09+0.067+0.0554+0.02+0.1+0.06+0.0349+0.0936) = 0.5205$$

$$\eta_{02} = (0.125+0.1385+0.096+0.1+0.04+0.0067+0.0593+0.0276) = 0.5930$$

$$\eta_{03} = (0.1356+0.0692+0.0533+0.0533+0.0333+0.0183+0.0299+0.0378) = 0.4108$$

$$\eta_{04} = (0.27+0.2+0.16+0.075+0.04+0.00977+0.09+0.0199) = 0.8647$$

This value of Grey relation is the overall performance that the enterprise requires.

Obtaining the Solar Panel Ranking

Because of $\eta_{04} > \eta_{02} > \eta_{01} > \eta_{03}$, the ranking order of candidate solar panels is: (I) solar panel 4; (II) solar panel 2; (III) solar panel 1; (IV) solar panel 3. Solar panel 4 is the optimal selection if both qualitative and quantitative criteria are taken into account. It is noted that the ranking order will change with respect to change in weighting value for each criterion. In other words, the customer may select a suitable solar panel based on his own requirements.

In order to simplify the evaluation process of AHP and GRA, a program written in EXCEL has been developed to simplify the calculation procedure. The authors had discussion with evaluators; they all accept that the integrated model is better than the current approaches and feasible by using the AHP-GRA decision-aiding software. Through the proposed model, it is possible to effectively integrate the specialized knowledge and experience of dispersed each evaluator, and the quantitative data to select a best solar panel to cooperation. It is found from the case study results that the GRA method is flexible method to the solar panel selection problem when both qualitative and quantitative criteria are to be taken into consideration.

CONCLUSIONS

Solar panel selection is a complex multi-criteria decision problem in nature that needs to include both qualitative and quantitative factors. In this study, an integrated model of AHP and GRA has been designed and applied to examine its

feasibility in selecting a best solar panel. This methodology will significantly reduce the purchasing cost and increase the production efficiency and overall competitiveness. This integrated model has more flexibility than the existing solar panel selection methods. AHP method is used to reflect the weights of qualitative criteria and integrate the various expectations from different evaluators into evaluating the solar panels. This integrated model is best suited to deal with multi-criteria decisions that involve both qualitative and quantitative factors, and application of the GRA method enables the selection of a best solar panel based on limited information and data. The decision-support system has been developed to conduct the solar panel selection process very effectively and to save the time for mathematical calculations.

It can be concluded that the integrated model can be widely applied for the selection of solar panels. In comparison with other models, this integrated model is user friendly and effective. There are several methods for evaluating multiple attributes. These can all be applied to the solar panel selection problem. Further research may be concentrated on the application of these methods and a comparison of the relative effectiveness of the results.

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Appendix. Algorithm of Grey Relational Analysis

The algorithm of GRA is described as follows (Wu and Chen, 1999):

Step 1

Generate the referential series of $x_0 = (x_0(1), x_0(2), \dots, x_0(j), \dots, x_0(n))$, in which x_i is the compared series of $(x_i(1), x_i(2), \dots, x_i(j), \dots, x_i(n))$, where $i = 1, 2, \dots, m$. The compared series x_i can be represented in a matrix form.

Step 2

Normalize the data set. The series data can be treated using one of the following three types: "larger-is-better", "smaller-is-better", and "nominal-is-best". For "larger-is-better" data transformation, $x_i(j)$ can be transformed into

$x_i^*(j)$. The formula is defined as in equation (1):

$$x_i^*(j) = \frac{x_i(j) - \min_i x_i(j)}{\max_j x_i(j) - \min_j x_i(j)} \quad (1)$$

Where, $\max_j x_i(j)$ is the maximum value of entity j and $\min_j x_i(j)$ is the minimum value of entity j .

For “smaller-is-better”, $x_i(j)$ can be transformed into $x_i^*(j)$ by equation (2):

$$x_i^*(j) = \frac{\max_j x_i(j) - x_i(j)}{\max_j x_i(j) - \min_j x_i(j)} \quad (2)$$

Therefore, the normalized referential series of x_0 becomes x_0^* . The original data set needs to be normalized with one of the types of data transformations.

Step 3

Calculate the distance of $\Delta_{0i}(j)$ – that is, the absolute value of the difference between x_0^* and x_i^* at the point of j .

Step 4

Calculate the grey relational coefficient $\beta_{0i}(j)$ using the equation (3):

$$\beta_{0i}(j) = \frac{\Delta \min + \phi \Delta \max}{\Delta_{0i}(j) + \phi \Delta \max} \quad (3)$$

Where, $\Delta \min = \min_i \min_j \Delta_{0i}(j)$, $\Delta \max = \max_i \max_j \Delta_{0i}(j)$ and ϕ is the distinguished coefficient ($\phi \in [0,1]$).

Step 5

Calculate the degree of the grey equation coefficient η_{0i} . If the weights (W_i) of criteria are determined, the grey relational grade is defined as follows

$$\eta_{0i} = \sum_{j=1}^n [W_i(j) \times \beta_{0i}(j)] \quad (4)$$